British Informatics Olympiad Final

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Dining Philosophers

A group of n philosophers (numbered from 0 to $n-1$) are sitting, in order, around a circular table. Between every pair of adjacent philosophers is a fork — to the left of philosopher i is fork i and fork $(i+1)$ **mod** n is to the right. [a **mod** b is equivalent to the remainder when a is divided by b.] In the middle of the table sits a communal bowl of spaghetti. The philosophers get hungry and, being rather clumsy eaters, require two forks to eat.

Philosopher i moves through four states. In state 1 (thinking) no forks are held. To move to state 2 (anticipation), they must pick up fork i (to their left). To move to state 3 (eating), they must pick up fork $(i+1)$ **mod** n (to their right). To move to state 4 (digesting), fork $(i+1)$ **mod** n is put down to the right. Finally, to return to state 1, they put down fork i to the left. Forks cannot be held by more than one philosopher at a time so a philosopher may have to wait before moving to a new state. Philosophers change their minds (and their states) regularly, and will not stay in any state for an excessive amount of time if they are able to change.

Question 1

Suppose there are 10 philosophers and 6 forks are currently picked up. What is the largest number of philosophers who may be eating? What is the smallest number?

Question 2

If there are n philosophers sitting around the table there are 4^n different combinations of states for the philosophers but, due to the shared forks not all of these combinations are possible. If there are 2 philosophers, how many obtainable combinations are there? How about if there are 3 philosophers?

Suppose all of the philosophers are thinking, get hungry and pick up the fork to their left. Each philosopher now requires the fork to their right but, in every case, this fork is being held by another philosopher. This situation is called *deadlock*. No further state changes are possible and no philosopher will ever be able to eat.

Question 3

Are there any other situations in which philosopher i might get hungry but be prevented from eating? Justify your answer.

We can try to deal with deadlock, either by preventing it or having a method for dealing with it when it arises.

Question 4

Suppose philosopher 0 has just had such an interesting idea that they never wish to leave the thinking state. Can a deadlock arise? Justify your answer.

Let $F^r_{x,y}$ be the number of different obtainable state combinations philosophers 1 to r (inclusive) can be in, where philosopher 1 is in state x and philosopher r is in state y.

Question 5

For $r \leq n-2$, give expressions for $F^{r+1}_{1,1}$, $F^{r+1}_{1,2}$, $F^{r+1}_{1,3}$ and $F^{r+1}_{1,4}$ using the $F^r_{x,y}$ values. Hence, or otherwise, briefly outline an algorithm for calculating the number of obtainable state combinations for n philosophers.

Not only is it desirable for the combination of philosophers to avoid deadlock, but it is also important that a philosopher who wishes to eat will eventually be able to eat.

Question 6

For each of the following methods you should say whether it avoids deadlock, whether a philosopher who wishes to eat will eventually be able to eat, and if there are any other side effects:

1. There is an extra fork in the centre of table, which any philosopher can reach and use instead of either of their adjacent forks.
2. Philosophers may drop a fork before eating, ie. they can move from anticipation to thinking.
3. Odd numbered philosophers take their left fork then their right fork, even numbered philosophers take their right fork then their left fork.

An alternative method for avoiding deadlock is for an external force, say a waiter, to place restrictions on the philosophers. In such a situation, a philosopher would indicate that he is hungry, but would not be allowed to pick up a fork until told to by the waiter.

Question 7

Outline a rule for a waiter which prevents deadlock and ensures a philosopher who wishes to eat will eventually be able to eat. Justify your answer.

Individual philosophers are unable to tell if the system is deadlocked, since they are only aware of their own state and the availability of the forks. If they are waiting for a fork they do not know whether or not the system in deadlocked.

Question 8

Consider the original rules, and suppose you are able to glance towards the dining philosophers but, getting a partial view of the table, can only see philosopher 0. Illustrate how the system appears to behave from your external view.

Question 9

In the general case, each philosopher may be different, in both the number of states and the items of cutlery to be taken or replaced when moving between states. Given a list, indicating how each philosopher moves between their respective states, outline an algorithm for determining if the combined system can deadlock.